Lecture 4, 10/05/12

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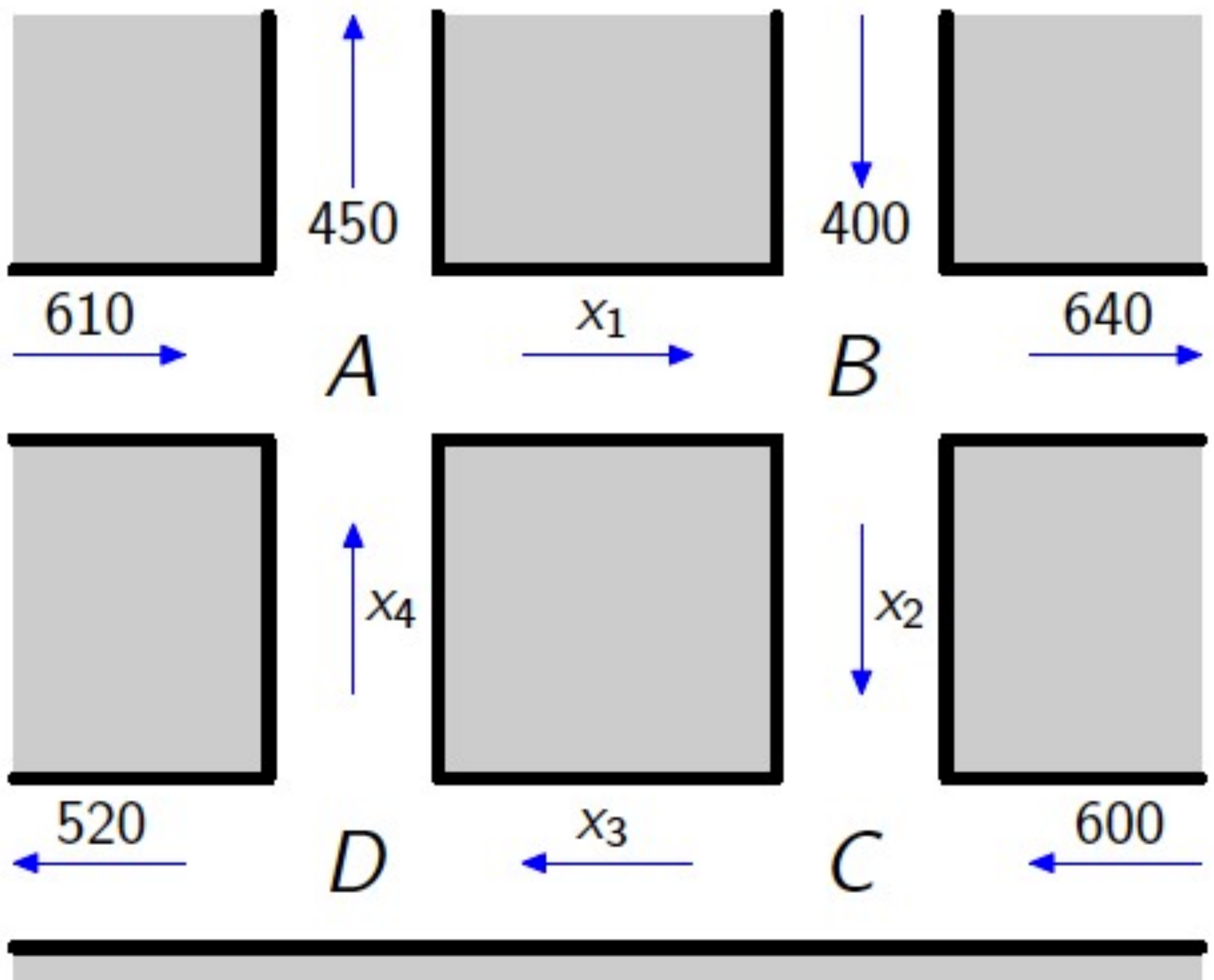
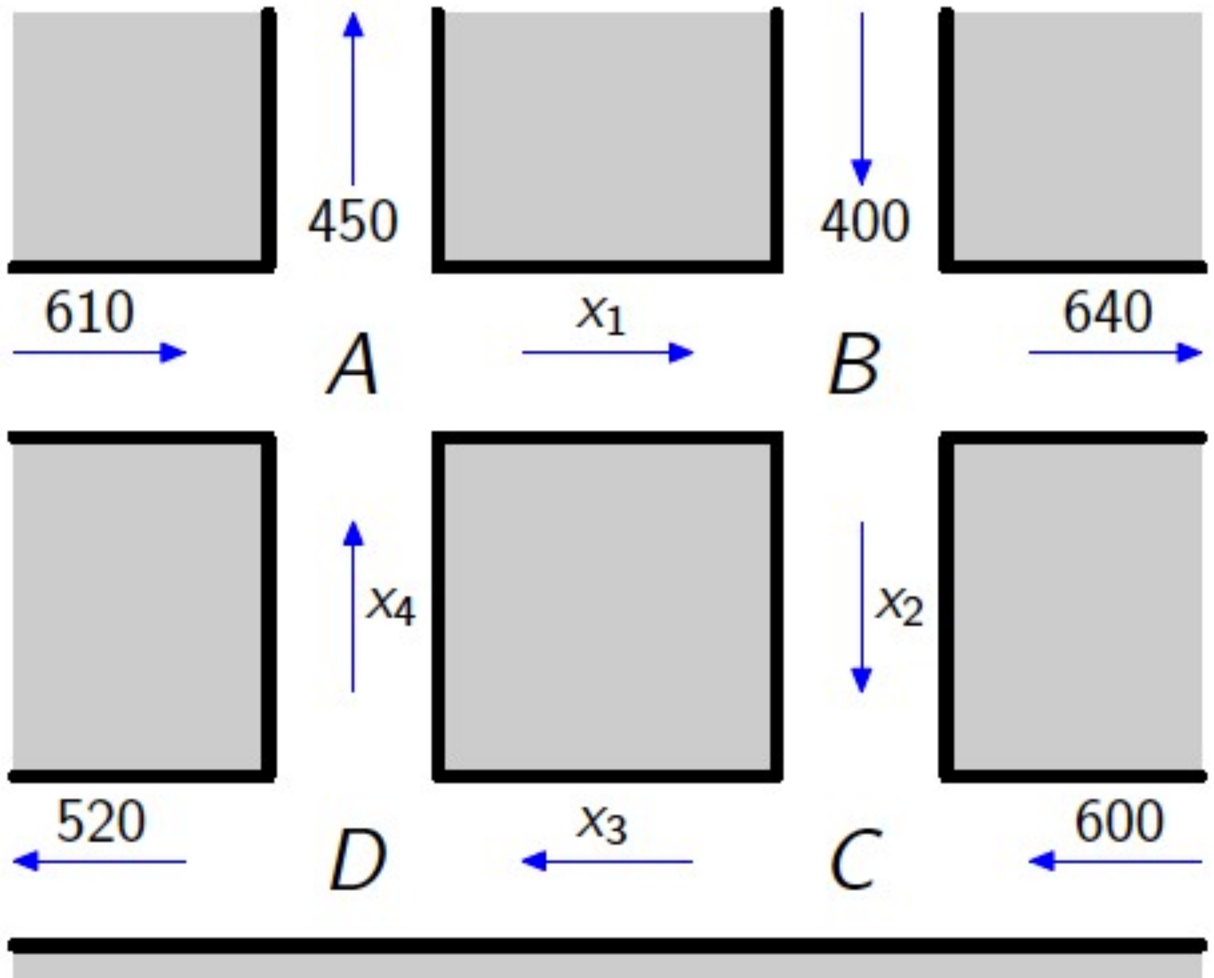
Linear System Example



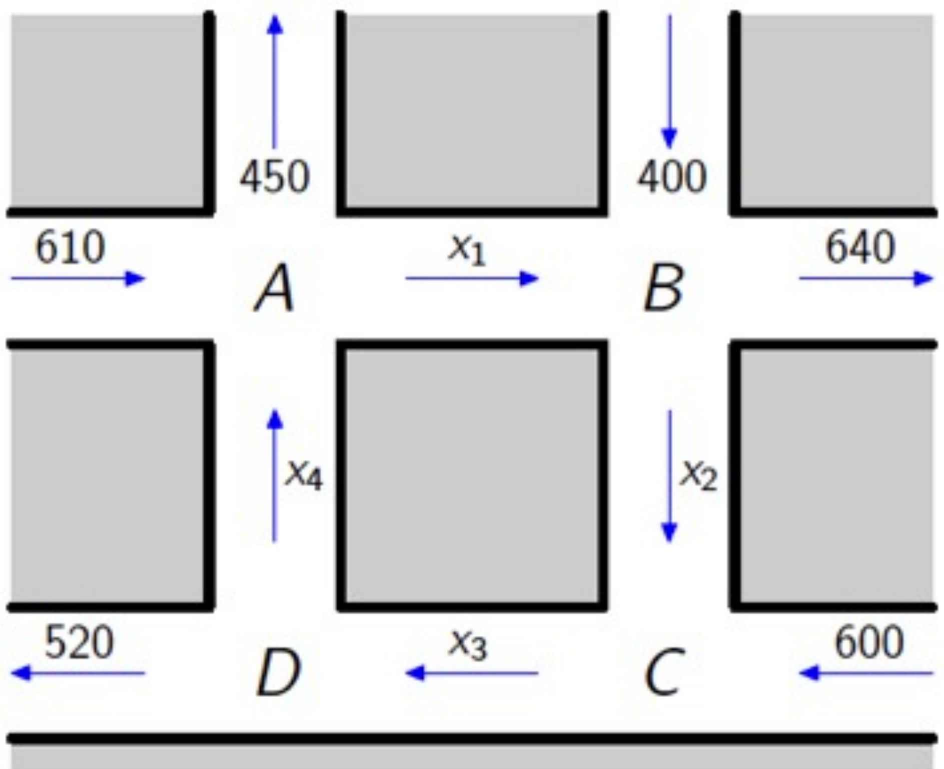
- Compute the traffic flow

Linear System Example



- The sum of the cars entering and leaving each intersection must be 0!

Linear System Example



A: $x_4 + 610 - 450 - x_1 = 0$

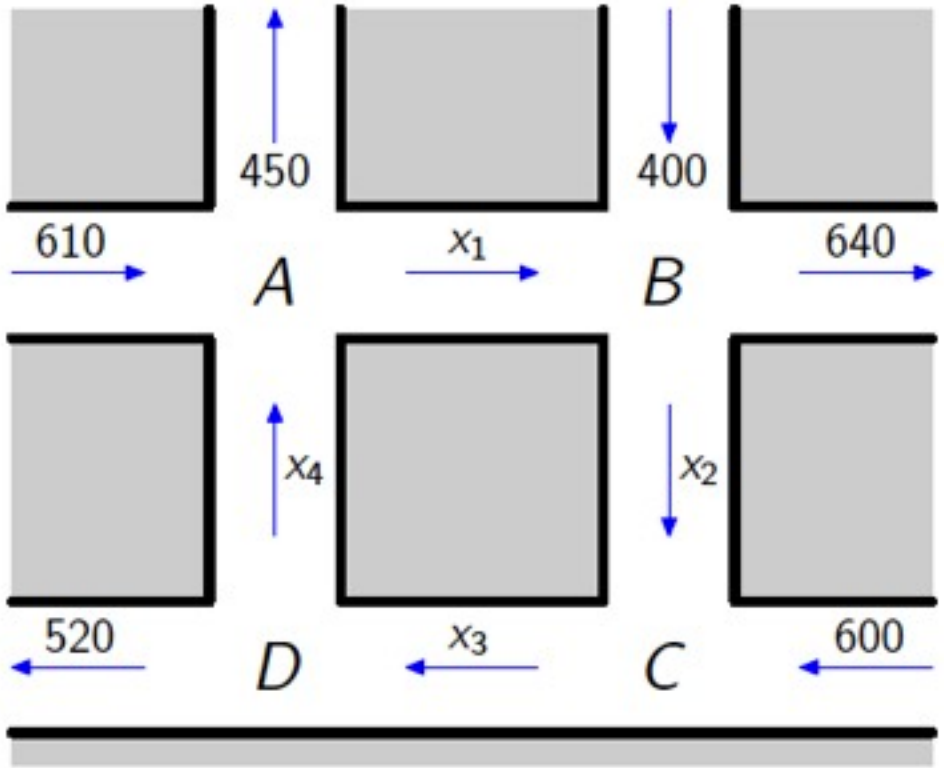
B: $x_1 + 400 - 640 - x_2 = 0$

C: $x_2 + 600 - x_3 = 0$

D: $x_3 - 520 - x_4 = 0$

- The sum of the cars entering and leaving each intersection must be 0!

Linear System Example



A: $-x_1 + x_4 = -160$

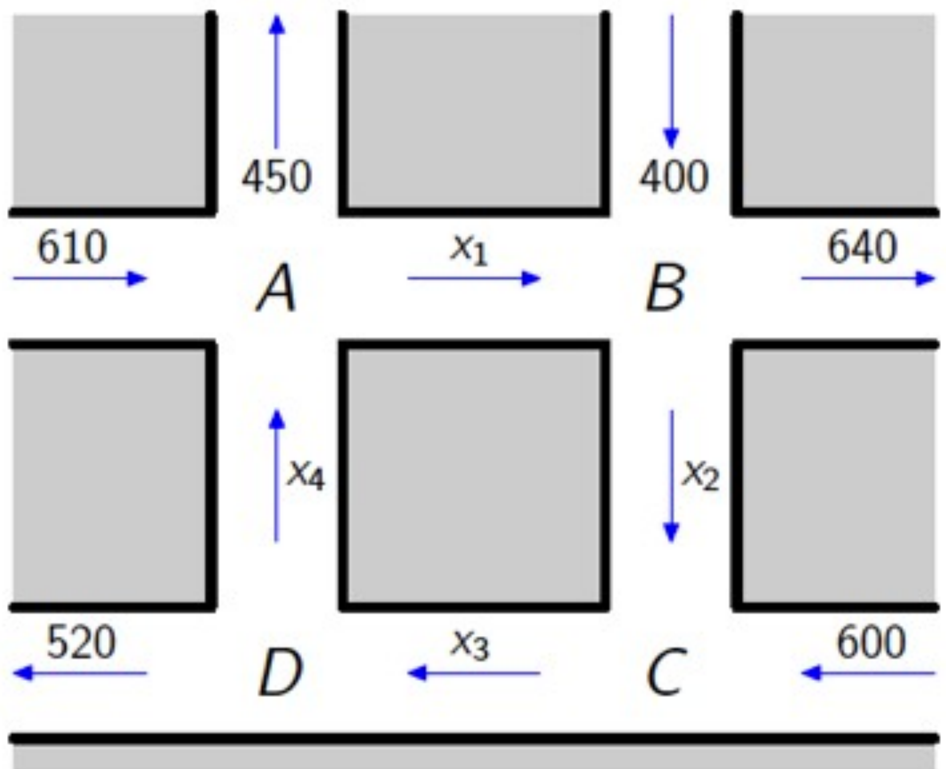
B: $x_1 - x_2 = 240$

C: $x_2 - x_3 = -600$

D: $x_3 - x_4 = 520$

- 4 unknowns and 4 pieces of information (equations). So it is likely (but not guaranteed) there will be one unique

Linear System Example

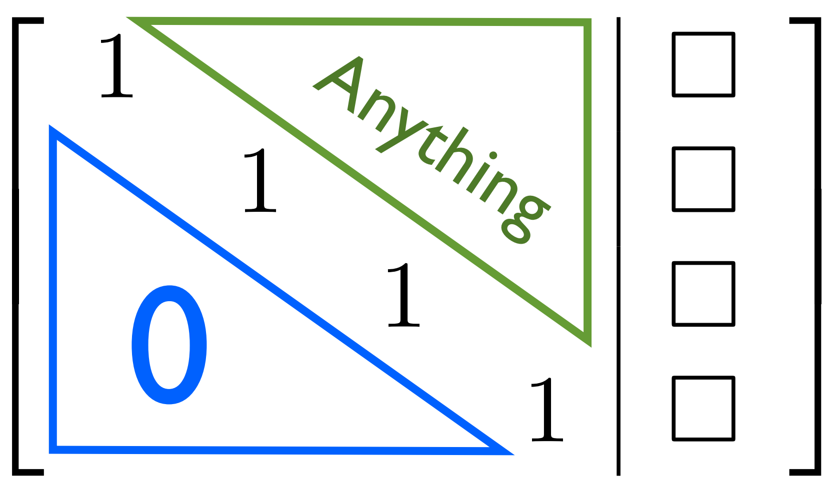


$$\left[\begin{array}{cccc|c} -1 & 0 & 0 & 1 & -160 \\ 1 & -1 & 0 & 0 & 240 \\ 0 & 1 & -1 & 0 & -600 \\ 0 & 0 & 1 & -1 & 520 \end{array}\right]$$

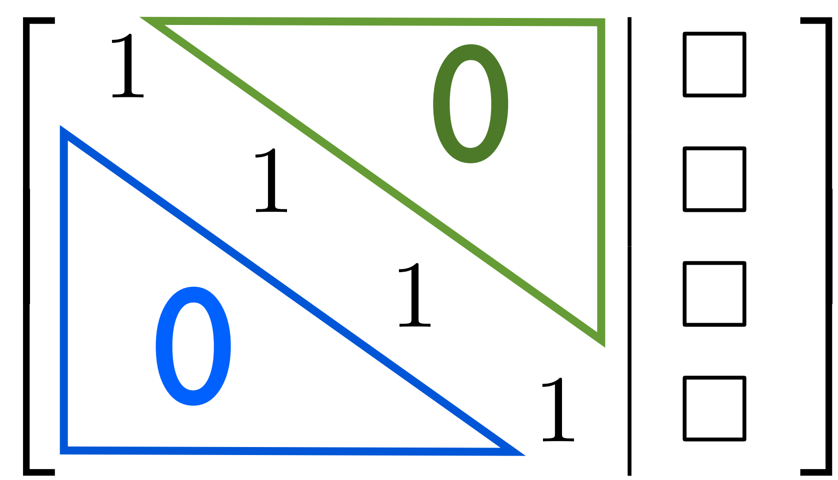
- Row reduce and solve.

Linear system example

Row Echelon Form



Reduced Row Echelon Form



Matrix equation method

- An alternative method for solving systems of equations.
- Rewrite the problem as an equation of sorts for which we can develop an arithmetic to solve the problem.

$$A \cdot x = b$$

To do this...

- We will first describe how to reformulate the linear system.
- Then we will develop a new kind of arithmetic (i.e. multiplication, addition, division, and more) to deal this reformulation.

Terminology

- A ***matrix*** is a rectangular array of numbers or symbols arranged in rows or columns.
- A ***vector*** is a matrix with either 1 row or one column.
  - If it has a single row, it's called a ***row vector.***
  - If it has a single column, it is called a ***column vector.***

Examples

Matrix

$$\begin{bmatrix} -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

Column vector

$$\begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Row vector

$$\begin{bmatrix} -1 & 0 & 0 & 1 \end{bmatrix}$$

- A vector is really just a special case of a matrix.
  - Row vector is a 1 x n matrix.
  - Column vector is a m x 1 matrix.

Goal

- Given a system of equations, we want to reformulate the problem as
  - $A \cdot x = b$
- Then we can say $x = \dfrac{b}{A}$ after suitably defining division.

Vector form of a linear equation.

- Any linear equation can be rewritten as a vector equation.

$$-x_1 + 3x_2 + 4x_3 = 5$$

$$\downarrow$$

$$\begin{bmatrix} -1 & 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 5$$

$$A \qquad \vec{x} \qquad \vec{b}$$

Notation and Conventions

- A - is referred to as the coefficient matrix.
- $\vec{\square}$ - is a notation indicating a vector
- $\vec{x}$ - is called the vector of unknowns
- $\vec{b}$ - is called the constant vector

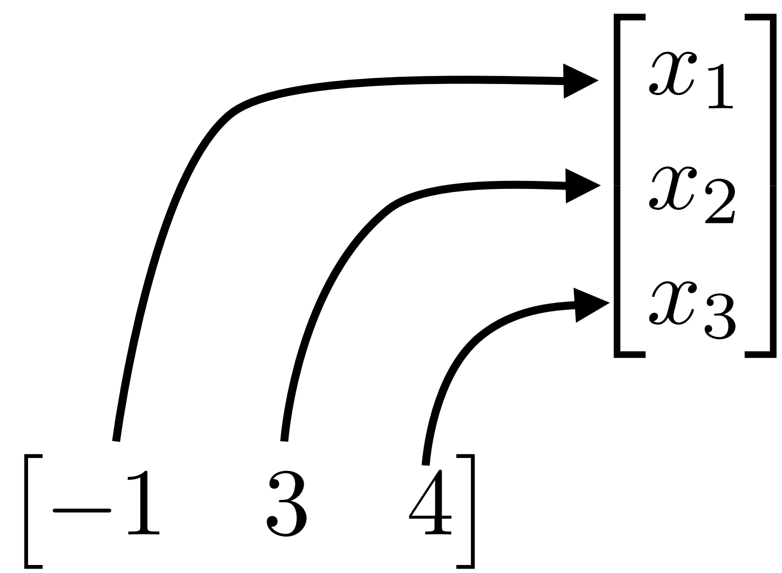
Define what we mean by multiplication

$$A \cdot \vec{x}$$

Definition of Multiply

- Continued on board.

$$\begin{bmatrix} -1 & 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$



$$\begin{bmatrix} -1 \\ 3 \\ 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$-1x_1 + 3x_2 + 4x_3$$

Important Points about Matrix Multiplication

1. Not all matrices can be multiplied!
2. Sometimes $A \cdot B$ makes sense but $B \cdot A$ does not!
3. Even if $A \cdot B$ and $B \cdot A$ both make sense, usually $A \cdot B \neq B \cdot A$