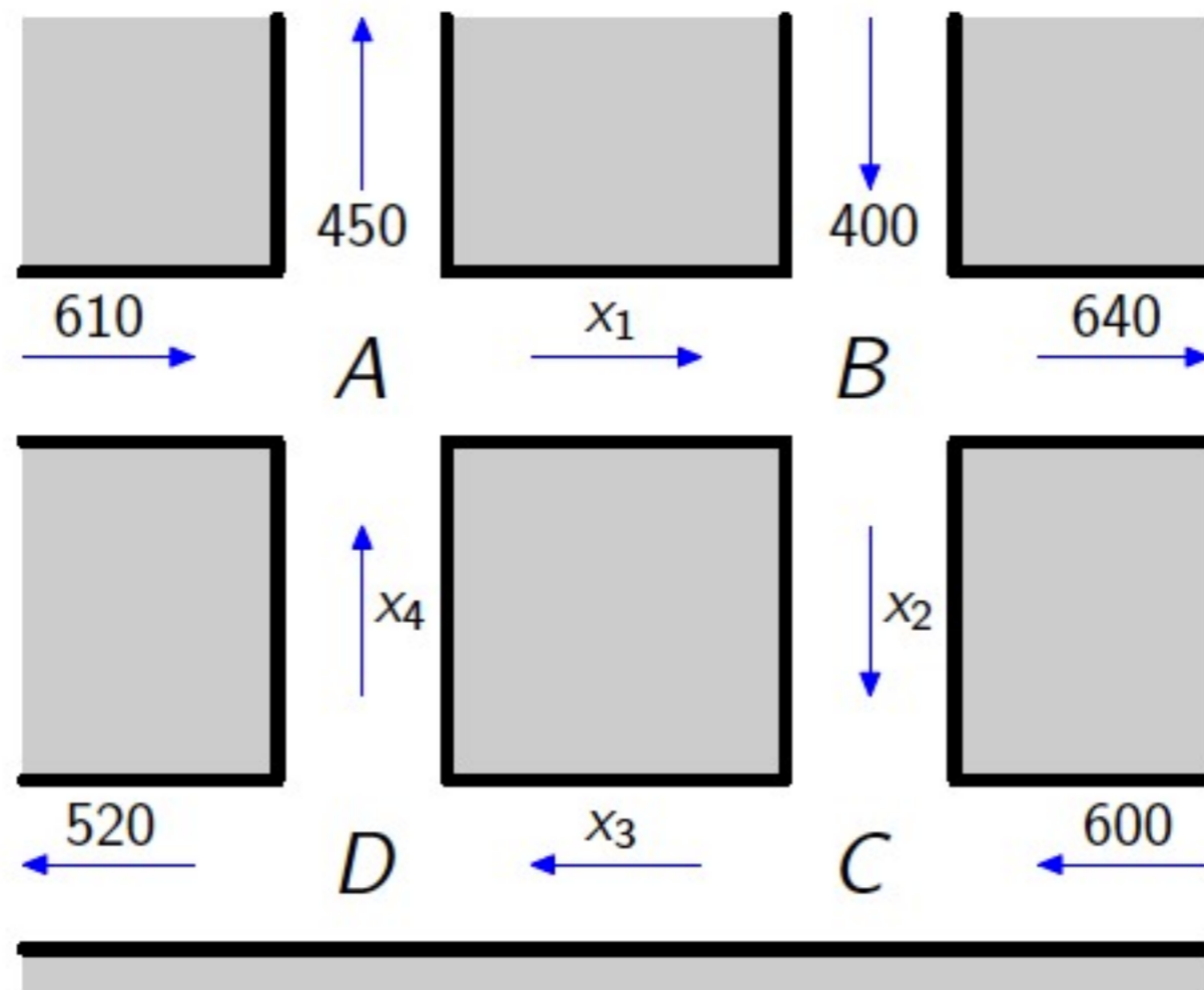


# Lecture 4, 10/05/12

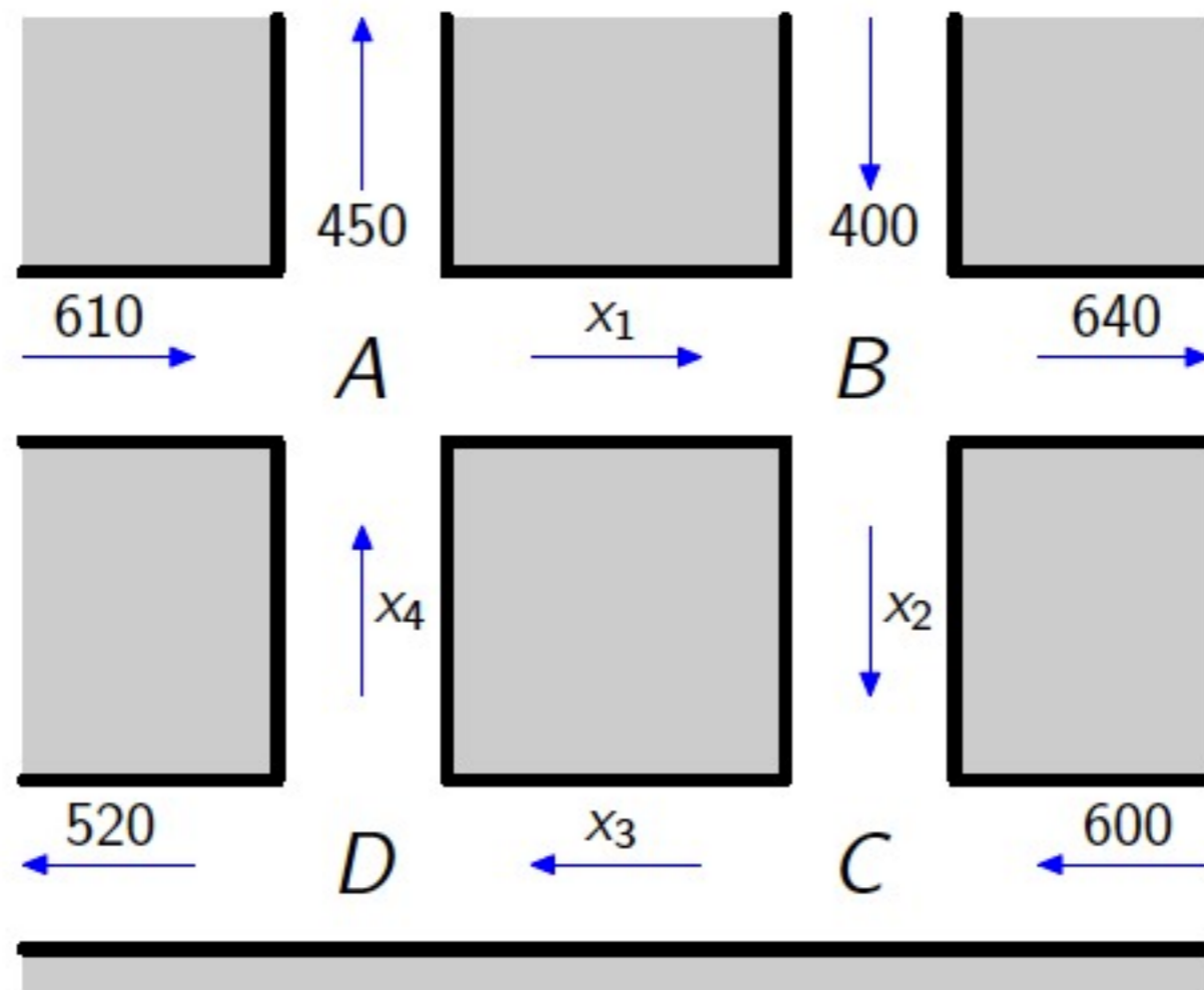
William Holmes

# Linear System Example



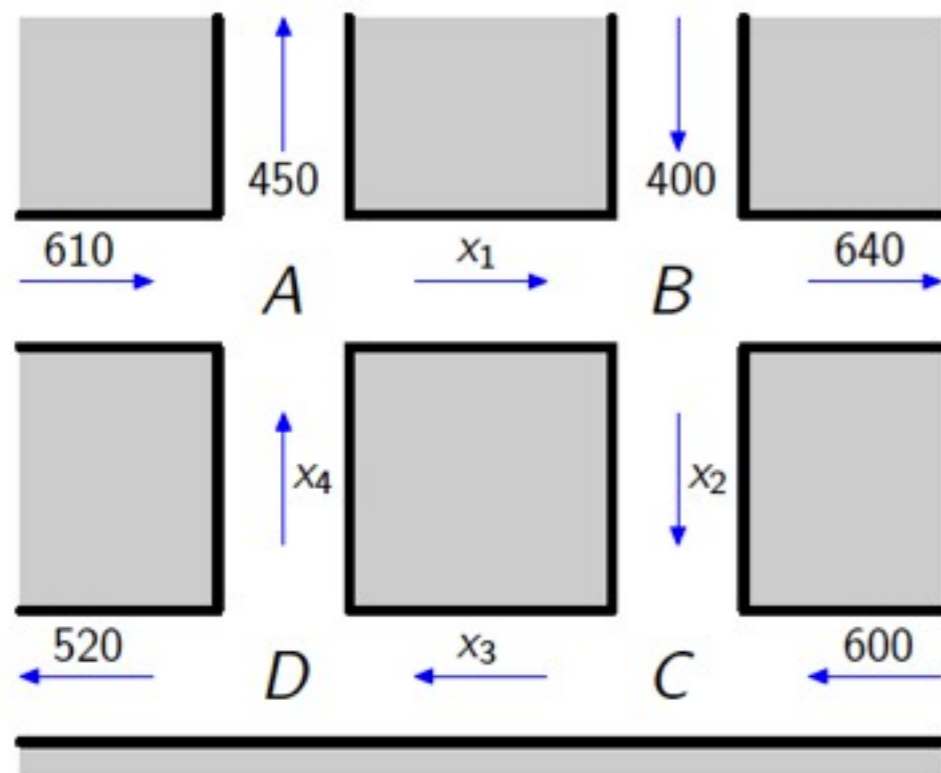
- Compute the traffic flow

# Linear System Example



- The sum of the cars entering and leaving each intersection must be 0!

# Linear System Example



$$\text{A: } x_4 + 610 - 450 - x_1 = 0$$

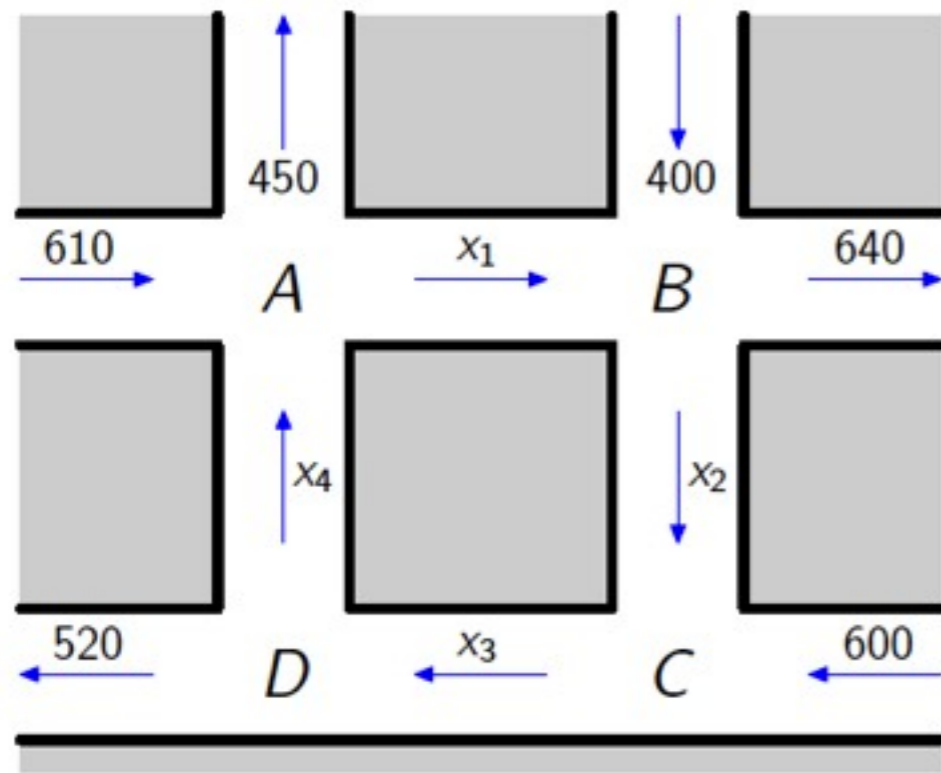
$$\text{B: } x_1 + 400 - 640 - x_2 = 0$$

$$\text{C: } x_2 + 600 - x_3 = 0$$

$$\text{D: } x_3 - 520 - x_4 = 0$$

- The sum of the cars entering and leaving each intersection must be 0!

# Linear System Example



$$A: -x_1 + x_4 = -160$$

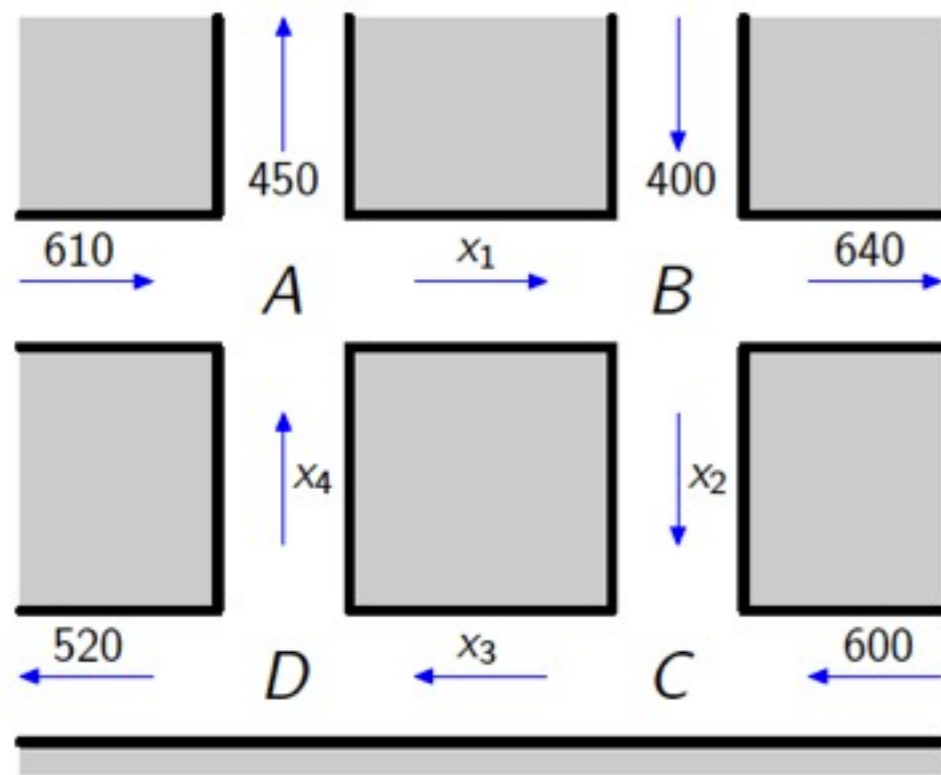
$$B: x_1 - x_2 = 240$$

$$C: x_2 - x_3 = -600$$

$$D: x_3 - x_4 = 520$$

- 4 unknowns and 4 pieces of information (equations). So it is likely (but not guaranteed) there will be one unique

# Linear System Example



$$\left[ \begin{array}{cccc|c} -1 & 0 & 0 & 1 & -160 \\ 1 & -1 & 0 & 0 & 240 \\ 0 & 1 & -1 & 0 & -600 \\ 0 & 0 & 1 & -1 & 520 \end{array} \right]$$

- Row reduce and solve.

# Linear system example

Row Echelon Form

$$\left[ \begin{array}{ccc|c} 1 & & & \square \\ & 1 & & \square \\ & & 1 & \square \\ & & & \square \end{array} \right]$$

A 4x4 augmented matrix in Row Echelon Form. The matrix is enclosed in large square brackets. The first three columns are enclosed in a green triangle pointing downwards from the top-left to the bottom-right. The word "Anything" is written in green inside this triangle. The fourth column contains four empty square boxes. The first row has a '1' in the first column. The second row has a '1' in the second column. The third row has a '1' in the third column. The fourth row has a '1' in the fourth column. A large blue '0' is located in the bottom-left corner of the matrix.

Reduced Row Echelon Form

$$\left[ \begin{array}{ccc|c} 1 & & 0 & \square \\ & 1 & & \square \\ & & 1 & \square \\ & & & \square \end{array} \right]$$

A 4x4 augmented matrix in Reduced Row Echelon Form. The matrix is enclosed in large square brackets. The first three columns are enclosed in a green triangle pointing downwards from the top-left to the bottom-right. The fourth column contains four empty square boxes. The first row has a '1' in the first column and a '0' in the third column. The second row has a '1' in the second column. The third row has a '1' in the third column. The fourth row has a '1' in the fourth column. A large blue '0' is located in the bottom-left corner of the matrix.

# Matrix equation method

- An alternative method for solving systems of equations.
- Rewrite the problem as an equation of sorts for which we can develop an arithmetic to solve the problem.

$$A \cdot x = b$$



# To do this...

- We will first describe how to reformulate the linear system.
- Then we will develop a new kind of arithmetic (i.e. multiplication, addition, division, and more) to deal with this reformulation.

# Terminology

- A **matrix** is a rectangular array of numbers or symbols arranged in rows or columns.
- A **vector** is a matrix with either 1 row or one column.
  - If it has a single row, it's called a **row vector**.
  - If it has a single column, it is called a **column vector**.

# Examples

Matrix

Column  
vector

Row  
vector

$$\begin{bmatrix} -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$[-1 \quad 0 \quad 0 \quad 1]$$

- A vector is really just a special case of a matrix.
- Row vector is a  $1 \times n$  matrix.
- Column vector is a  $m \times 1$  matrix.


# Goal

- Given a system of equations, we want to reformulate the problem as
  - $A \cdot x = b$
  - Then we can say  $x = \frac{b}{A}$  after suitably defining division.

# Vector form of a linear equation.

- Any linear equation can be rewritten as a vector equation.

$$-x_1 + 3x_2 + 4x_3 = 5$$


$$\begin{matrix} [-1 & 3 & 4] \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 5 \\ A & \vec{x} & \vec{b} \end{matrix}$$

# Notation and Conventions

- $A$  - is referred to as the coefficient matrix.
- $\vec{\square}$  - is a notation indicating a vector
- $\vec{x}$  - is called the vector of unknowns
- $\vec{b}$  - is called the constant vector

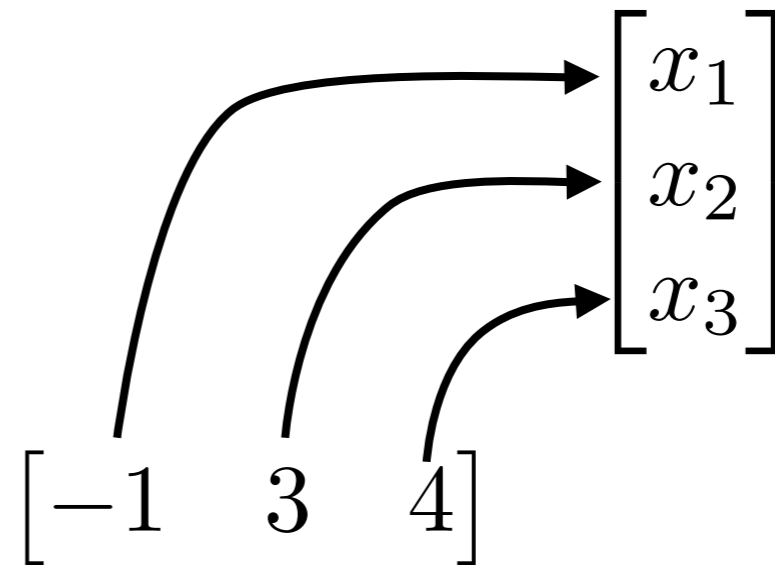
Define what we mean  
by multiplication

$$A \cdot \vec{x}$$

# Definition of Multiply

- Continued on board.

$$\begin{bmatrix} -1 & 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$



$$\begin{bmatrix} -1 \\ 3 \\ 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$-1x_1 + 3x_2 + 4x_3$$



# Important Points about Matrix Multiplication

1. Not all matrices can be multiplied!
2. Sometimes  $A \cdot B$  makes sense but  $B \cdot A$  does not!
3. Even if  $A \cdot B$  and  $B \cdot A$  both make sense, usually  $A \cdot B \neq B \cdot A$